MATHEMATICS

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AIM POINT
MATHEMATICS
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TRIGONOMETRIC RATIO AND IDENTITIES

& Their Properties

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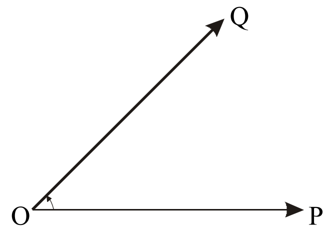
THINGS TO REMEMBER

✱ Measure of Angles :

The measure of angle is the amount of rotation from the direction of one ray of the angle to the other. The initial and final positions of the revolving ray are respectively called the initial and terminal sides (arms). Also, the revolving line is called the generating side. eg, if initial and final positions of the revolving ray are OP and OQ, then the angle formed will be ∠POQ.

If the rotation is in clockwise sense, the angle measured is negative and if the rotation is in anti-clock-wise sense, the angle measured is positive.



System of Measurement of Angles :

There are three system of mesurement

(i) Sexagesimal System

(ii) Centesimal System

(iii) Circular system

(i) Sexagesimal System : In this system each angle is divided into 90 equal parts and each part is known as a degree. Thus a right angle is equal to 90 degree. One degree is denoted as 1^o.

Each degree is divided into 60 equal parts each of which is known as one minute. One minute is denoted as 1'. Each minute is consist of 60 parts, each part is known as a second. One second is denoted by 1".

Hence, 1 right angle = 90^o (90 degree)

1 degree = 60' (60 minute)

1 minute = 60" (60 second)

(ii) Centesimal System : In this system each angle is divied into 100 equal parts and one part is known as a grade. Thus one right angle is equal to 100 grade. One grade is denoted as 1^g.

One grade is divided into 100 equal parts, one part is known as a minute and is denoted as 1'.

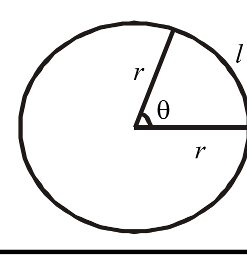
One minute is also divided into 100 euqal parts, one part is known as a second which is denoted by 1".

Hence, 1 right angle = 100^g (100 grade)

1 grade = 1^g = 100' (100 minute)

1 minute = 1' = 100" (100 second)

(iii) Circular system : If the angle subtended by an arc of length l to the center of circle of radius r, is θ then $\theta = \frac{l}{r}$



If the length of arc is equal to the radius of the circle, then the angle subtended at the center of the circle will be one raidan. One radian is denoted by 1^c.

The ratio of the circumference of the circle to the diameter of the circle is denoted by a greek letter π and it is a constant quentity.

$$\therefore \qquad \frac{\text{Circumference of circle}}{\text{Diameter of circle}} = \pi$$

This constant quantity π is a irrational quentity and generally its approximate value is $\frac{22}{7}$ and its general value upto six place of dicimal is 3.142857.

Relation Among Degree, Radian and Grade :

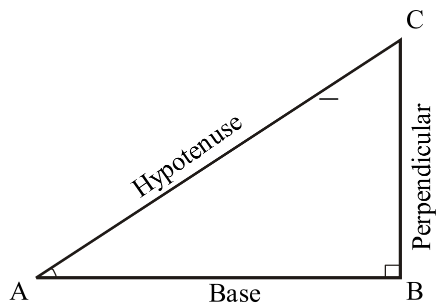
The number of radians of an angle subtended by an arc of a circle at the center is equal to the ratios of arc and radius.

$$180^o = \pi^c = 100^g$$

and $$1 \text{ radian} = 57^o 17' 44.8''$$

✷ **Trigonometric Ratios :**

Let us take a right angled triangle ABC right anled B.



Let ∠CAB = θ, then

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} \qquad \text{Cosec}\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{BC}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC}$$

Except these isx ratios, there are two other terms which are known as 'versed sine' and 'coversed sine' respectively.

ie, $$\text{ver} \sin\theta = 1 - \cos\theta$$

and $$\text{cover} \sin\theta = 1 - \sin\theta$$

✷ **Fundamental Relation Among Trigonometric Ratios :**

It is clear from the definitions of trigonometric rations that

1. $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} \Rightarrow \sin\theta \operatorname{cosec}\theta = 1$

2. $\sec\theta = \dfrac{1}{\cos\theta} \Rightarrow \cos\theta \sec\theta = 1$

3. $\cot\theta = \dfrac{1}{\tan\theta} \Rightarrow \tan\theta \cot\theta = 1$

4. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

5. $\cos^2\theta + \sin^2\theta = 1$

6. (a) $1 + \tan^2\theta = \sec^2\theta$

   (b) $\sec^2\theta - \tan^2\theta = 1$

7. (a) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

   (b) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

Representation of a Trigonometric Ratio in Any Other Trigonometric Ratios :

	sin θ	**cos θ**	**tan θ**	**cot θ**	**sec θ**	**cosec θ**
sin θ	sin θ	$\sqrt{(1-\cos^2\theta)}$	$\dfrac{\tan\theta}{\sqrt{(1+\tan^2\theta)}}$	$\dfrac{1}{\sqrt{(1+\cot^2\theta)}}$	$\dfrac{\sqrt{(\sec^2\theta-1)}}{\sec\theta}$	$\dfrac{1}{\operatorname{cosec}\theta}$
cos θ	$\sqrt{(1-\sin^2\theta)}$	cos θ	$\dfrac{1}{\sqrt{(1+\tan^2\theta)}}$	$\dfrac{\cot\theta}{\sqrt{(1+\cot^2\theta)}}$	$\dfrac{1}{\sec\theta}$	$\dfrac{\sqrt{(\operatorname{cosec}^2\theta-1)}}{\operatorname{cosec}\theta}$
tan θ	$\dfrac{\sin\theta}{\sqrt{(1-\sin^2\theta)}}$	$\dfrac{\sqrt{(1-\cos^2\theta)}}{\cos\theta}$	tan θ	$\dfrac{1}{\cot\theta}$	$\sqrt{(\sec^2\theta-1)}$	$\dfrac{1}{\sqrt{(\operatorname{cosec}^2\theta-1)}}$
cot θ	$\dfrac{\sqrt{(1-\sin^2\theta)}}{\sin\theta}$	$\dfrac{\cos\theta}{\sqrt{(1-\cos^2\theta)}}$	$\dfrac{1}{\tan\theta}$	cot θ	$\dfrac{1}{\sqrt{(\sec^2\theta-1)}}$	$\sqrt{(\operatorname{cosec}^2\theta-1)}$
sec θ	$\dfrac{1}{\sqrt{(1-\sin^2\theta)}}$	$\dfrac{1}{\cos\theta}$	$\sqrt{(1+\tan^2\theta)}$	$\dfrac{\sqrt{(1+\cot^2\theta)}}{\cot\theta}$	sec θ	$\dfrac{\operatorname{cosec}\theta}{\sqrt{(\operatorname{cosec}^2\theta-1)}}$
cosec θ	$\dfrac{1}{\sin\theta}$	$\dfrac{1}{\sqrt{(1-\cos^2\theta)}}$	$\dfrac{\sqrt{(1+\tan^2\theta)}}{\tan\theta}$	$\sqrt{(1+\cot^2\theta)}$	$\dfrac{\sec\theta}{\sqrt{(\sec^2\theta-1)}}$	cosecθ

✱ **Complementary and Supplementary Angles :**

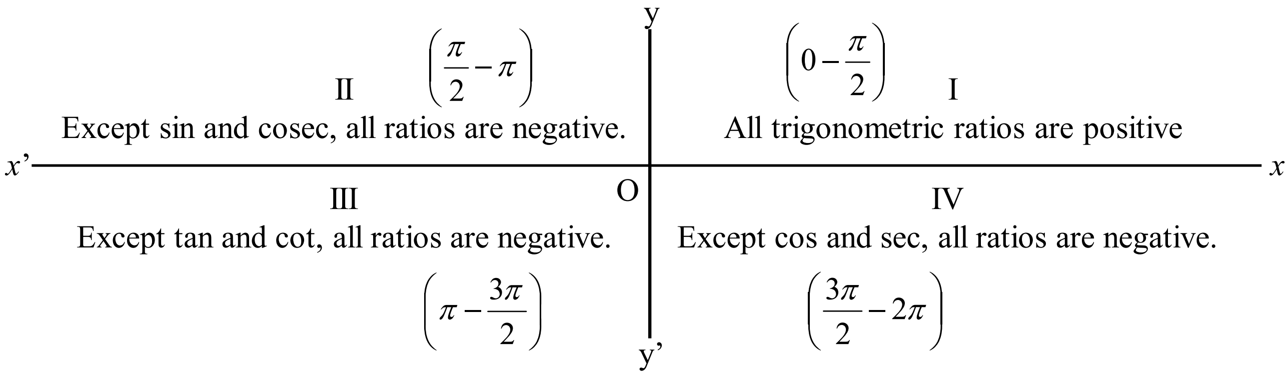
If the sum of two angles is equal to right angle, then these angles are known as complementary angles of each other. Thus, θ and 90° - θ are complementary angles of each other.

Now, if the sum of two angles is equal to two right angles, then these angles are known as supplementry angles of each other. Thus, θ and 180° - θ are supplementary angles of each other.

eg, 23° and 67° are complementary angles of each other while 167° and 13° are supplementary angles of each other.

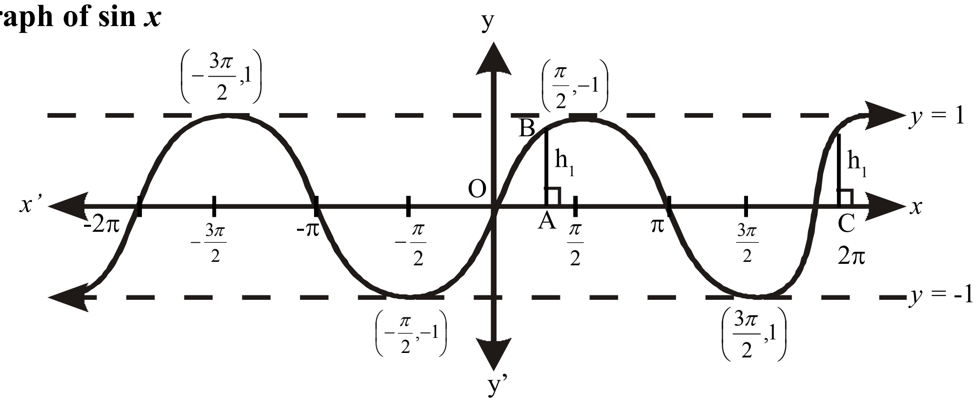
Trigonometric Ratio of Complementary and supplementary Angles :

α	sinα	cosα	tanα	cotα	secα	cosecα
- θ	-sinθ	cosθ	-tanθ	-cotθ	secθ	-cosecθ
90° - θ	cosθ	sinθ	cosθ	tanθ	cosecθ	sinθ
90° + θ	cosθ	-sinθ	-cotθ	-tanθ	-cosecθ	secθ
180° - θ	sinθ	-cosθ	-tanθ	-cotθ	-secθ	cosecθ
180° + θ	-sinθ	-cosθ	tanθ	cotθ	-secθ	-cosecθ



✷ Graph of Trigonometric Function :

1. Graph of sin x



Facts Related to sin x.

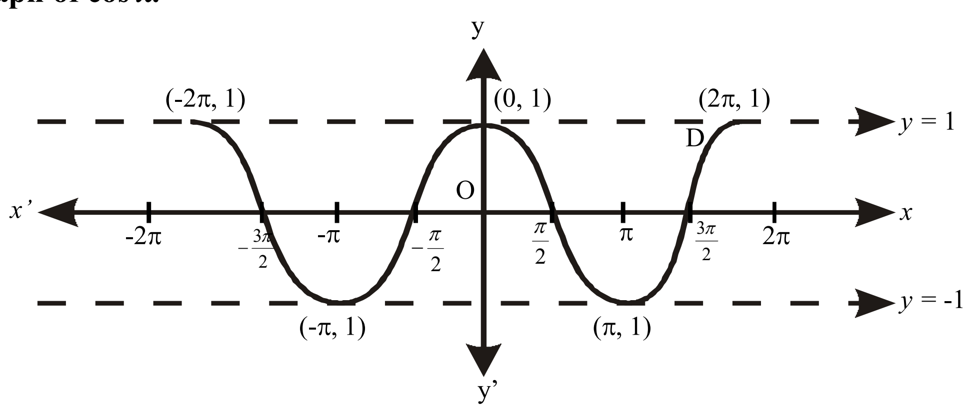
(a) Domain = R

(b) Range = [-1,1]

(c) Period = 2π

(d) Graph of sin x is continuous for all real values of x.

2. Graph of cos x.



Facts Related to cos x.

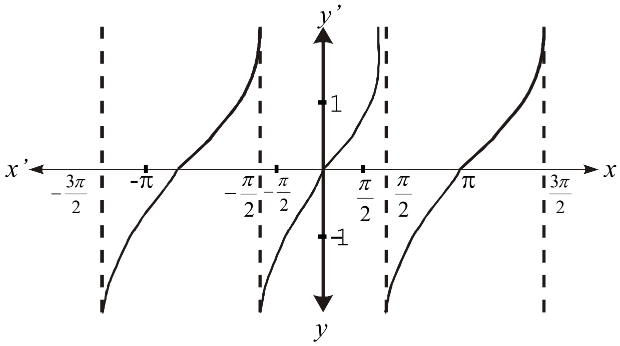
(a) Domain = R

(b) Range = [-1,1]

(c) Period = 2π

(d) Graph of cos x is continuous for all real values of x.

3. Graph of tan *x*.



Facts Related to tan *x*.

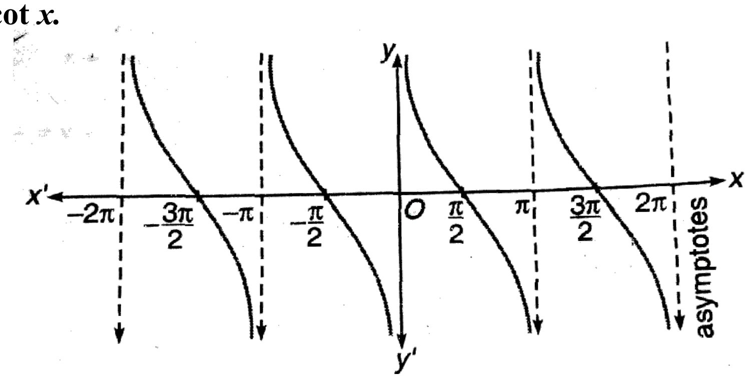
(a) Domain = R ~ $(2n + 1)\frac{\pi}{2}$, n ∈ *I*

(b) Range = [-∞, ∞]

(c) Period = π

(d) Graph of tan *x* is discontinuous at $x = \frac{m\pi}{2}$, where *m* is an, odd integer.

4. Graph of cot *x*.



Facts Related to tan *x*.

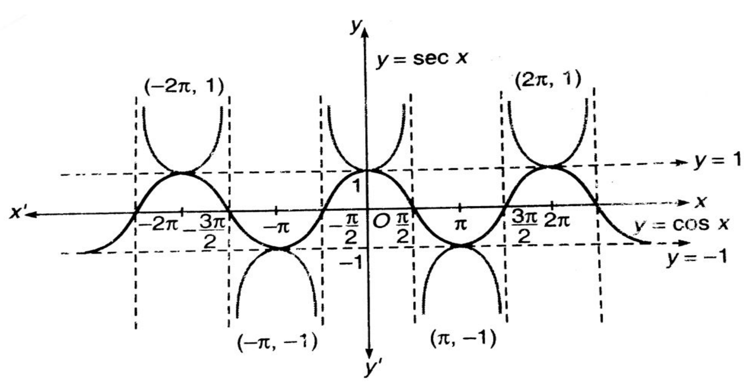
(a) Domain = R ~ *n*π, *n* ∈ *I*

(b) Range = [-∞, ∞]

(c) Period = π

(d) Graph of cot *x* is discontinuous at *x* = *m*π, where *m* is an integer.

5. Graph of sec *x*.



Facts Related to sec *x*.

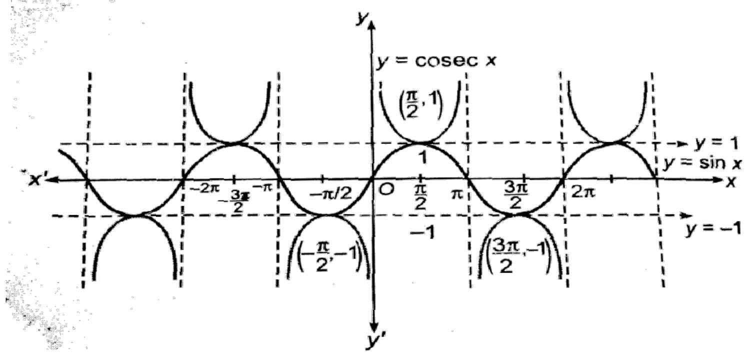
(a) Domain = R ~ $(2n + 1)\frac{\pi}{2}$, n $\in I$

(b) Range = $(-\infty, -1] \cup [1, \infty)$

(c) Period = π

(d) Graph of sec *x* is discontinuous at $x = \frac{m\pi}{2}$, where *m* is an, odd integer.

5. Graph of cosec *x*.



Facts Related to cosec *x*.

(a) Domain = R ~ $n\pi$, $n \in I$

(b) Range = $(-\infty, -1] \cup [1, \infty)$

(c) Period = 2π

(d) Graph of cosec *x* is discontinuous at $x = m\pi$, where *m* is an integer.

Note :

- If measure of an angle is given in degree, then to convert it into radians, multiply the measure of an angle by $\frac{\pi}{180^o}$ and if the measure of an angle is given in radians, then to convert it into degree, write 180^o at the place of π
- $\sin(n\pi + (-1)^n\theta) = \sin\theta$, n $\in I$
- $\cos(2n\pi \pm \theta) = \cos\theta$, n $\in I$
- $\sin(n\pi + \theta) = \tan\theta$, n $\in I$